The Capacitance of a Small Circular Schottky Diode for Submillimeter Wavelengths

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Abstract— **The capacitance of a small-area circular submillimeter wave diode is strongly affected by the edge effect of the charged anode. The correction factor due to the edge effect cannot be obtained analytically and the capacitance of a circular diode must be calculated using numerical methods. In this work a new, numerically derived formula for the junction capacitance of a small circular diode is presented.**

I. Introduction

SCHOTTKY diodes have been used for several decades as a standard nonlinear component in mixers and frequency multipliers at millimeter wavelengths, and the equivalent circuit of the diode has been widely studied [1]. The capacitance-voltage characteristic of the diode is well known; a correction factor for the edge-effects has typically been included in the model of the capacitance [2], [3]. However, at submillimeter wavelengths the radius of a circular Schottky diode is so small that the edge effects should be studied more carefully. The edge effect can be solved analytically only for some simple spherical geometries [4]. Therefore, numerical methods are needed to find an exact solution that satisfies boundary conditions and is also physically valid in the semiconductor.

II. Formulation of the Problem

A circular flat metallic anode is assumed to be at the top of the epitaxial semiconductor, as shown in Fig. 1. The radius of the anode is R_0 and the origin of the cylindrical coordinate system is in the center of the circular anode at the semiconductor-metal interface. The approximate shape of the depletion layer is shown in Fig. 1 when the anode is charged to a potential $\phi_0 = V - \phi_{\mathrm{bi}}$, where ϕ_{bi} is the built-in potential and V is an external voltage.

The net charge of the depletion layer can be found by solving the potential ϕ and the electric field E $(= -\nabla\phi)$ in the semiconductor by using Poisson's equation

$$\nabla^2\phi = -\rho/\epsilon, \tag{1}$$

where ϵ is the permittivity and ρ is the net volume charge given by [5]

$$\rho = qN_d\left(1 - e^{q\phi/kT}\right), \tag{2}$$

where q is the charge of the electron, k is Boltzmann's constant, N_d is the doping density, and T is the temperature.

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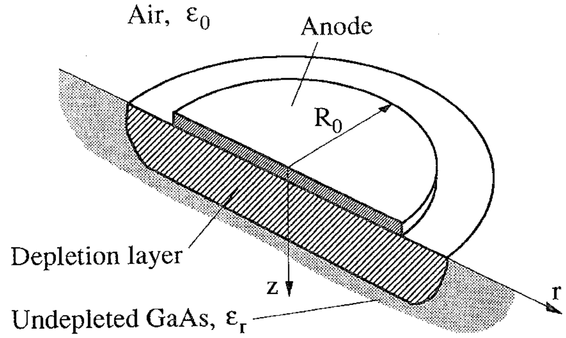

Fig. 1. Schematic of a circular Schottky diode.

Futhermore, at the air-semiconductor interface the potential and the tangential component of the electric field must be continuous. The potential ϕ, which satisfies (1) and (2) and the boundary conditions, cannot be obtained analytically [3]. However, in a one-dimensional case $(R_0 \gg W)$ the width of the depletion layer is $W = \sqrt{-2\phi_0\epsilon/qN_d}$.

III. Numerical Method

The potential ϕ and the electric field E in the semiconductor can be determined numerically by using finite difference method, as Wasserstrom and McKenna have done [3] for the case of a rectangular anode. The semiconductor is divided into a dense mesh, size of $N \times M$. The potential of each mesh point must satisfy (1) and (2). In a cylindrical coordinate system, the Laplacian is given by the five-point formula [6]

$$\nabla^2\phi_{m,n} = \frac{1}{h^2}\Bigg\{ -4\phi_{m,n} + \phi_{m,n+1} + \phi_{m,n-1} + \left[1 - \frac{h}{2r_m}\right]\phi_{m-1,n} + \left[1 + \frac{h}{2r_m}\right]\phi_{m+1,n}\Bigg\}, \tag{3}$$

where $\phi_{m,n}$ is the potential at a mesh point (r_m, z_n) and h is the mesh spacing. The potential in the semiconductor can now be found by solving $N \times M$ nonlinear equations. Because the number of equations is very large, the simplest way to solve the potential is to use an iterative over-relaxation method [7]. At the air-semiconductor interface, the potential is determined by using boundary conditions—continuous potential and continuous tangential component of the electric field. The potential in the air is found by using Green's formula [3].

Numerical results for the potential and field of a typical submillimeter wave varactor $(R_0/W \approx 2)$ have been plotted in Fig. 2. Although the solution of the potential is well-behaved, the electric field has a singularity near the edge of the anode. This singularity can be understood, because near the edge

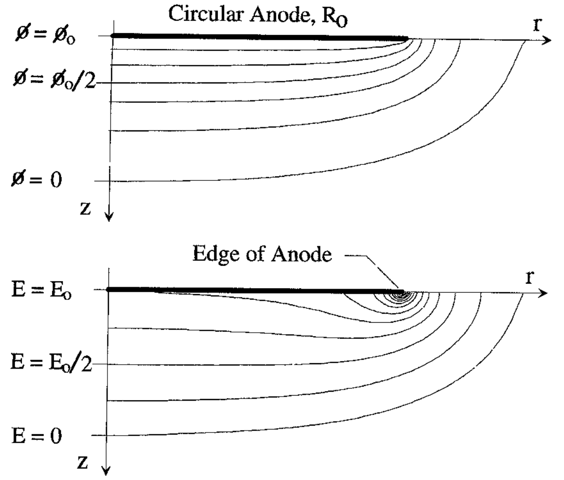

Fig. 2. Schematic of the equipotential curves (upper) and curves of constant field amplitude.

the anode looks like a sharp wedge and the field includes $1/d^n$ components, where d is the distance from the edge of the anode. The singularity is only mathematical, because the microstructure of the real diode affects the electric field so that it cannot be infinite. However, the difference between the numerical solution and the real solution does not affect the solution of the potential.

IV. Capacitance of the Diode

The capacitance of a diode is

$$C = \frac{\partial Q}{\partial \phi_0}, \tag{4}$$

where the net charge of the depletion layer is

$$Q = \int_0^\infty \int_0^{2\pi} \int_{-\infty}^\infty \rho(r, \varphi, z)\ r\ dr\ d\varphi\ dz. \tag{5}$$

This integral is computed by evaluating the sum

$$Q = \sum_{n=1}^{N} \sum_{m=1}^{M} \rho_{m,n} 2\pi r_m h^2. \tag{6}$$

As already noticed by Wasserstrom and McKenna [3], the actual shape of the depletion layer is almost independent of the potential ϕ_0 for a large $|\phi_0|$. Therefore, the net charge can be written with a good accuracy as

$$Q = qN_d W^3 \left[\pi \frac{R_0^2}{W^2} + 2\pi D_1 \frac{R_0}{W} + D_2 \right], \tag{7}$$

when $W < R_0$. The first term in (7) is the charge, which is obtained if the edge effects are ignored. D_1 and D_2 are correction terms, which can be found by fitting (7) to the numerical results of (6). From (4) and (7), the capacitance of the diode can be found to be

$$C = \frac{\epsilon \pi R_0^2}{W} \left[1 + 4D_1 \frac{W}{R_0} + \frac{3D_2}{\pi} \frac{W^2}{R_0^2} \right], \tag{8}$$

$$= \frac{\epsilon \pi R_0^2}{W} \gamma, \tag{9}$$

where γ is the correction factor due to the edge effects. The second term in (8) is the standard, first order correction term

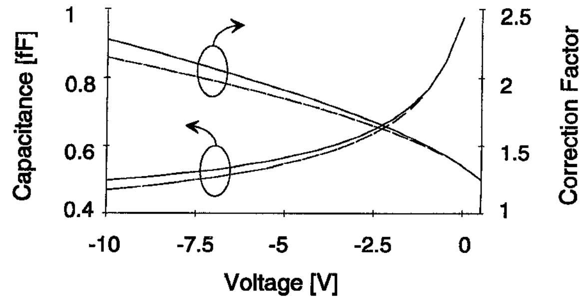

Fig. 3. Capacitance C and correction factor γ, when $N_d = 1 \cdot 10^{17}$ 1/cm^3 and $R_0 = 0.5\ \mu$m. Solid line is calculated with the new formula and dashed line with the formula obtained by Copeland.

[2]. The last term in (8) is the second order correction term, which takes into account the edge effects due to the circular nature of the anode.

The potential in the semiconductor has been numerically calculated for a large range of R_0/W ratios. The calculated net charge from equation (6) has been fitted with equation (7) and the following result for GaAs has been obtained

$$D_1 = 0.36 \quad \text{and} \quad D_2 = 0.34,$$

which is valid for $W < R_0$. In the case of a large anode ($R_0 \gg W$) the result agrees with the result obtained by Copeland [2], because the last term in (8) is negligible. However, for a small anode, the second order correction term is not negligible as shown in Fig. 3.

V. Conclusion

A new formula for the capacitance of the circular diode has been derived. The new correction term is so small that for typical diodes used at micro and millimeter wavelengths the formula derived by Copeland can be used [2]. However, for very small submillimeter wave varactors the extra, second order correction term is not negligible and the new, more accurate formula should be employed.

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